

Primordial Non-Gaussianities of General Multiple Field Inflation

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ABSTRACT: We perform a general study of the primordial scalar non-Gaussianities in multi-field inflationary models in Einstein gravity. We consider models governed by a Lagrangian which is a general function of the scalar fields and their first-order spacetime derivatives. We use δN formalism to relate scalar fields and the curvature perturbations. We calculate the explicit cubic-order perturbative action and the three-point function of curvature perturbation evaluated at the horizon-crossing. Under reasonable assumptions, in the small slow-varying parameters limit and with a sound speed c_s close to one, we find that the non-Gaussianity is completely determined by these slow-varying parameters and some other parameters which are determined by the structure of the inflationary model. Our work generalizes previous results, and implies the possibility of the existence of large non-Gaussianity in model constructing, and it would be also useful to study the non-Gaussianity in multi-field inflationary models which will be constructed in the future.

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1. Introduction

Inflation has been a very successful paradigm for understanding the evolution of the very early universe [1]. It not only naturally provides a way to solve flatness and horizon problems, but also generates density perturbations as seeds for the large-scale structure in the universe. Inflation is most commonly discussed in terms of a potential energy which is a function of a single, slowly rolling scalar field. Such models generically predict an almost scale-invariant spectrum and an almost Gaussian distribution of adiabatic density perturbations on super-Hubble scales [2]. These generic predictions are consistent with recent cosmological observations of the temperature anisotropy and polarization of the Cosmic Microwave Background [3].

The measurement of primordial perturbations provides increasingly precise determination of the spectrum index n_s and tensor-to-scalar ratio r , which come from the power spectrum, in other words, the two-point correlation function of the curvature perturbation. However, there are many alternative models of inflation are able to give the similar predictions, and thus there is still considerable ambiguity in constructing the real inflationary model.

In contrast, the non-Gaussian component of the scalar perturbations is characterized by the correlation functions beyond two-point, e.g. the three-point function of the fluctuations, which is a nontrivial function of three variables and will provide us more information beyond the power spectrum. Furthermore, as described above, the non-Gaussianity of distribution of primordial fluctuations predicted by the simplest model of inflation is well below the current limit of measurement [6, 7, 8, 9]. Therefore, any detection of large non-Gaussianity would be a significant challenge to our current understanding of the early universe.

Indeed, there is the possibility of the presence of large non-Gaussianity with $f_{\text{NL}} \gg 1$ [3, 4]. The latest observational bound on the three-point function of the primordial curvature perturbation ζ from the WMAP 5-year data states that the local non-gaussianity parameter f_{NL} is limited to the values $-9 < f_{\text{NL}}^{\text{local}} < 111$ [3]. In the next few years, with improved experiments like the Planck satellite, we will measure the CMB anisotropies to an incredible resolution at $|f_{\text{NL}}| < 5$.

On the theoretical side, the level of non-Gaussianities can be generally calculated analytically in typical single or multiple-field inflationary models (see [5] for a review). The three-point function for the standard single-field inflation models with a canonical kinetic term was performed in an elegant and gauge invariant way in [6, 7], and the corresponding f_{NL} is $\mathcal{O}(10^{-2})$ and too small to be detected. In order to find possibly detectable large non-Gaussianities in single-field cases, models with non-standard actions have also been studied [21, 22, 26, 27, 71].

In the past few years, intensive efforts have been devoted to connecting string theory and inflation [10, 11, 12, 13, 14]. Indeed, in models descending from the low-energy limit of superstring theory or string compactification, there are many light moduli fields describing the higher-dimensional degrees of freedom, which could play a role during inflation. Therefore, it is interesting and of physical significance to generalize the above analysis of non-Gaussianities in single-field models to the multiple-field inflation models [29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47]. Most of the previous works focus on the models with standard canonical kinetic terms for the scalar fields involved. It is shown that in models satisfying the slow-roll conditions, f_{NL} is also of order slow-roll parameters as in the standard single-field case. Models with non-canonical kinetic terms have also been investigated, in particular the effective multiple-field DBI-inflation [45, 46, 47]. Models with non-trivial field space metric G_{IJ} have also been considered [40].

In these inflationary models inspired by string theory or other theory beyond the Standard Model, there are several significant features different from those in the standard single field models, where the Lagrangian is of the form $\mathcal{L} = \sqrt{-g} [-\frac{1}{2}(\partial\phi)^2 - V(\phi)] = \sqrt{-g}(X - V)$. Firstly, the scalar field potential is not necessarily the only degree of freedom in model building, in contrast to that in the standard single field models. Indeed, in these string inspired models, it is generally expected that deviation from the standard kinetic term $X \sim (\partial\phi)^2$ of the scalar field action would arise [15]. One would generally expect loop corrections in the quantum theory to generate operators in the Lagrangian that are proportional to higher-order derivatives X^2 , X^4 , and so on. Indeed, if the energy scale of renormalization is of order Planck scale M_{Pl} , such higher-order interactions would be suppressed and be negligible, and thus the canonical kinetic term would give a good approximation. On the other hand, if the inflation scale is close to the GUT scale, these ultraviolet corrections might be significant and of considerable relevance [16]. Such models with non-canonical kinetic terms have been considered previously by a number of authors [16, 17, 18, 19, 20]. For our purpose, the deviation from a canonical Lagrangian would be an important source of possible large non-Gaussianities.

Secondly, the presence of many scalar fields during inflation would affect the generation of primordial perturbations. In single field inflation, it is well-known that the curvature perturbation is conserved on super-horizon scales. However, in multi-field inflation models, the presence of multiple light fields will lead to the generation of non-adiabatic field perturbations during inflation. The curvature perturbation in multi-field inflation can generally evolve after Hubble exiting, due to the presence of entropy perturbations, which can be ‘sources’ of curvature perturbation. This non-trivial evolution of the overall curvature perturbation would be another source of detectable non-Gaussianity. A general description of calculating the three-point functions of the fluctuations in multi-field models was presented recently in [35], which emphasized this super-horizon evolution of curvature perturbation. It is also argued in [36] that in some cases, for example in the curvaton scenario [54, 55, 56, 57, 58, 59, 72, 61], there is the possibility that the large-scale superhorizon-effected components would dominate the pri-

mordial non-Gaussianity, rather than the components from the microscopic fluctuations. However, in the absence of any estimation for the microphysical contributions to the three-point functions, the above analysis is incomplete, and thus a precise investigation of non-Gaussianities generated from the fluctuations at the horizon-crossing is needed.

In this work, we consider a very large class of multiple-field inflation models, in which the dynamics of scalar fields is described by a Lagrangian $P(X, \phi^I)$, where P is an arbitrary function of \mathcal{N} scalar fields and their kinetic term $X = -\frac{1}{2}(\nabla\phi^I)^2$ [25]. This form of action includes the standard choice $P = X - V$ as a special case, and can be viewed as a generalization of the Lagrangian of k-inflation [16] to the cases of multiple scalar fields. Low-energy limit of string theory may also lead to such an action, such as Dirac-Born-Infeld action [23, 24], and its multi-field extension studied recently in [45, 46]. In general, one may consider theories including arbitrary number of higher-order derivatives in the action, such as $P(\phi, \partial\phi, \partial^2\phi, \dots)$. However, if the energy scale of inflation process is much lower than M_{pl} , the contributions from these higher-order derivatives will be suppressed and thus can be neglected. Therefore, we consider models where P contains only X . Due to the same reason, we consider Einstein gravity.

We assume that these scalar fields generate the density perturbations. We expand the general multi-field Lagrangian to cubic order of the perturbations $\delta\phi^I$, and use δN formalism [48, 49, 50, 51] to relate curvature perturbation ζ and these multiple scalar perturbations $\delta\phi^I$. We calculate the scalar three-point function of the curvature perturbations following Maldacena et. al. [6]. To control the calculation, we define some small slow-varying parameters, which can be viewed as the generalization of the standard slow-roll parameters. In our formalism, although the Lagrangian is very general, we assume the effective speed of sound c_s is almost close to one. In the limit of small slow-varying parameters, under some reasonable assumptions, we finally find that the non-Gaussianity in these models is completely characterized by the slow-varying parameters and some other parameters determined by the concrete structure of $P(X, \phi^I)$. In particular, our result shows the possibility of the presence of large non-Gaussianities, due to the deviation from the canonical Lagrangian.

The paper is organized as follows. In the next section, we setup the general multi-field inflationary models, and derive the background equations of motion. In order to control the calculation and analysis the solution, we define some slow-varying parameters in this general context. Then we give a brief review of calculating non-Gaussianities during multi-field inflation based on δN formalism. In section 3, we develop the second-order theory for the linear perturbations. We estimate the power spectrum in the limit of small slow-varying parameters. In section 4, we calculate the exact cubic perturbation action for this most general multi-field inflationary model. In section 5, we perform a general calculation of the three-point function, which represents the central result of this work. Finally, we make a conclusion in section 6.

We work in natural units, where $c = \hbar = M_{\text{pl}} = 1$, and $M_{\text{pl}} \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass.

2. Setup

2.1 Background

We consider a large class of multi-field inflation models, which are constructed from a generic set of \mathcal{N} scalar fields $\{\phi^I, I = 1, 2, \dots, \mathcal{N}\}$ coupled to Einstein gravity. The action takes the form:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2}R + P(X, \phi^I) \right], \quad (2.1)$$

with kinetic term

$$X = -\frac{1}{2}G_{IJ}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J, \quad (2.2)$$

where $g_{\mu\nu}$ is the spacetime metric with signature $(-, +, +, +)$ and $G_{IJ} = G_{IJ}(\phi)$ is the \mathcal{N} -dimensional field space metric, and P is an arbitrary function of X and ϕ 's.

The energy-momentum tensor of the scalar fields takes the form

$$T^{\mu\nu} = P g^{\mu\nu} + P_{,X} G_{IJ} \partial^\mu \phi^I \partial^\nu \phi^J, \quad (2.3)$$

where $P_{,X}$ denotes the partial derivative of P with respect to X . In order to consider the background (unperturbed) dynamics, we suppose that the universe is homogeneous, with a flat Friedmann-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t) dx^i dx^i, \quad (2.4)$$

where $a(t)$ is the scale factor and $H = \dot{a}/a$ is the Hubble parameter. Under this assumption, the energy-momentum tensor of the scalar fields reduces to that of a perfect fluid, with energy density

$$\rho = 2XP_{,X} - P, \quad (2.5)$$

and pressure $P = P(X, \phi^I)$.

The equations of motion for the scalar fields derived from (2.1) are [25]

$$\ddot{\phi}^I + \Gamma_{JK}^I \dot{\phi}^J \dot{\phi}^K + \left(3H + \frac{\dot{P}_{,X}}{P_{,X}} \right) \dot{\phi}^I - \frac{G^{IJ}}{P_{,X}} P_{,J} = 0, \quad (2.6)$$

where $P_{,I}$ denotes the derivative of P with respect to ϕ^I , and Γ_{JK}^I is the Christoffel symbols associated with the field space metric G_{IJ} . The equations of motion of the gravitational dynamics are Friedmann equation

$$H^2 = \frac{\rho}{3} \equiv \frac{1}{3}(2XP_{,X} - P), \quad (2.7)$$

and the continuity equation

$$\dot{\rho} = -3H(\rho + P) \equiv -6HXP_{,X}. \quad (2.8)$$

The combination of the above two equations gives another useful equation

$$\dot{H} = -XP_{,X}. \quad (2.9)$$

It is convenient to define an effective speed of sound c_s [16], as

$$c_s^2 \equiv \frac{P_{,X}}{\rho_{,X}} = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}. \quad (2.10)$$

Note that in models with canonical kinetic terms $P = X - V$, $c_s = 1$.

In general one may consider models with an arbitrary field space metric $G_{IJ}(\phi^I)$. In this work, we focus on the case where $G_{IJ} = \delta_{IJ}$ and thus $\Gamma_{JK}^I = 0$, i.e. the field space is flat. This choice of action already covers a large class of multi-field inflationary models with non-canonical kinetic terms. The linear perturbations of models with an arbitrary metric G_{IJ} have been investigated in [25].

2.2 Slow-varying parameters

For general function $P(X, \phi^I)$, it is difficult to solve the equations of motion for the scalars (2.6) analytically. In order to capture the main physical picture and investigate the evolution of the system, the idea is to define some small parameters to control the dynamics, and to find solutions perturbatively in power expansions of these small parameters.

In standard single field slow-roll inflation, this condition is achieved by assuming that the inflaton ϕ is rolling slowly in comparison with the expansion rate $|\dot{\phi}| \ll H$. Similarly, in general multi-field inflation models, it will prove convenient to introduce a dimensionless “slow-varying” matrix as [36]

$$\epsilon^{IJ} = \frac{P_{,X} \dot{\phi}^I \dot{\phi}^J}{2H^2} = \epsilon^I \epsilon^J, \quad (2.11)$$

where

$$\epsilon^I = \sqrt{\frac{P_{,X}}{2}} \frac{\dot{\phi}^I}{H}. \quad (2.12)$$

When there is only one inflaton field involved, ϵ^{IJ} reduces to the standard single field slow-roll parameter $\epsilon = -\dot{H}/H^2$, which can also be expressed as

$$\epsilon = \text{tr} \epsilon^{IJ} = G_{IJ} \epsilon^{IJ} = -\frac{\dot{H}}{H^2}. \quad (2.13)$$

It proves useful to decompose ϵ into two new parameters ϵ_X and ϵ_ϕ [26], which measure how the Hubble parameter H varies with the kinetic and potential parts of scalar fields ϕ^I respectively

$$\epsilon = \epsilon_X + \epsilon_\phi = -\frac{H_{,X}}{H^2} \dot{X} - \frac{H_{,I}}{H^2} \dot{\phi}^I. \quad (2.14)$$

As in the single field models where we may define $\eta = \frac{\dot{\epsilon}}{\epsilon H}$, here we define another slow-varying matrix η^{IJ} as

$$\eta^{IJ} \equiv \frac{\dot{\epsilon}^{IJ}}{\epsilon H}, \quad (2.15)$$

which can be written explicitly

$$\eta^{IJ} = 2\epsilon^{IJ} - \frac{P_{,X}(\ddot{\phi}^I \dot{\phi}^J + \dot{\phi}^I \ddot{\phi}^J) + \dot{P}_{,X} \dot{\phi}^I \dot{\phi}^J}{2H\dot{H}}. \quad (2.16)$$

The matrix η^{IJ} generalizes the slow-roll parameter η in single-field inflation models. Note that with a flat target space metric G_{IJ} , we have $\eta = \text{tr} \eta^{IJ} = \dot{\epsilon}/\epsilon H$ as expected.

Due to the generality of the function $P(X, \phi^I)$ and the complexity of the scalar equations of motion (2.6), the relations between $P_{,I}$ and the rolling of scalar fields $\dot{\phi}^I$ and $\ddot{\phi}^I$ are complicated. And thus we may introduce another set of parameters defined as

$$\tilde{\epsilon}_I = -\frac{P_{,I}}{3\sqrt{2P_{,X}}H^2}, \quad (2.17)$$

which can be viewed as the analogue of the standard slow-roll parameter of the form $\epsilon = \frac{1}{2}(V'/V)^2$. In models with standard canonical kinetic terms, it is easy to show that $|\tilde{\epsilon}^I| \approx |\epsilon^I|$ as expected, and thus two slow-roll parameters ϵ and η are enough to control the theory. However, for general function P , there is no simple relation between these two parameters. In order to proceed, we expect that both $\tilde{\epsilon}^I$ and ϵ^I are of the same order, and assume that $\dot{P}_{,X}/(HP_{,X})$ can be negligible. Therefore, from the scalar equations of motion (2.6), it is easy to see that $\tilde{\epsilon}^I \approx -\epsilon^I$ as expected. Similarly another matrix is defined as

$$\tilde{\eta}_{IJ} = -\frac{P_{,IJ}}{3H^2 P_{,X}}, \quad (2.18)$$

this is the analogue of $\eta = V''/V \approx V''/3H^2$ in the single field case.

Furthermore, we define the dimensionless parameters

$$u = \frac{1}{c_s^2} - 1, \quad s = \frac{\dot{c}_s}{c_s H}, \quad (2.19)$$

where u measures the deviation of the effective sound speed c_s from unity, and s measures the change speed of c_s . In models with canonical kinetic terms, $u = s = 0$.

These parameters generalize the usual slow-roll parameters, and in general depend on the kinetic terms as well as the potential terms. For generic theories we expect that $|\epsilon|, |u|, |s| \ll 1$ and¹

$$\epsilon^{IJ}, \eta^{IJ}, \tilde{\eta}^{IJ} \sim \mathcal{O}(\epsilon/\mathcal{N}) , \quad (2.20)$$

and thus $\epsilon^I \sim \tilde{\epsilon}^I \sim \mathcal{O}(\sqrt{\epsilon/\mathcal{N}})$, here \mathcal{N} is the number of the scalar fields. Furthermore, for those models with non-vanishing “cross” derivatives, i.e. $P_{,XI}, P_{,XXI}, P_{,XIJ} \neq 0$, we assume that the ‘ X ’-derivatives of these slow-varying parameters also satisfy some smallness relations

$$\tilde{\epsilon}_{,X}^I \sim \frac{P_{,X}}{H^2} \tilde{\epsilon}^I , \quad \tilde{\epsilon}_{,XX}^I \sim \frac{P_{,X}^2}{H^4} \tilde{\epsilon}^I , \quad \tilde{\eta}_{,X}^{IJ} \sim \frac{P_{,X}}{H^2} \tilde{\eta}^{IJ} . \quad (2.21)$$

In general, the validity of these conditions depends on the explicit forms of the models. In this work, we do not try to find explicit models which satisfy these slow-varying conditions. It is also addressed in [26, 27] and etc., in the presence of a non-canonical kinetic term, the smallness of these slow-varying parameters does not imply that the inflation itself is slow-rolling.

In order to simplify some of the derivations that follows, it will prove useful to define two parameters which are combinations of derivatives of P with respect to the kinetic term X ²

$$\Sigma = X P_{,X} + 2X^2 P_{,XX} = \frac{\epsilon H^2}{c_s^2} , \quad (2.22)$$

$$\lambda = X^2 P_{,XX} + \frac{2}{3} X^3 P_{,XXX} . \quad (2.23)$$

Here Σ is of order $\mathcal{O}(\epsilon)$, but for general $P(X, \phi^I)$ where $P_{,XI} \neq 0$, such as K-inflation or DBI-inflation, there is no simple relation between λ and the above slow-varying parameters.

2.3 δN Formalism

In this section we make a brief review of δN formalism [48, 49, 50, 51], which is proved to be a powerful technique to calculate the curvature perturbation in a variety of inflation models, especially in the multi-field models.

The idea of δN formalism is to identify primordial curvature perturbation ζ with the perturbation of the local expansion. Starting from a flat slice at some initial time t_i , the local expansion $N(t, t_i, \mathbf{x})$ at some final time t is defined as

$$N(t, t_i, \mathbf{x}) = \int_{t_i}^t dt' H(t', \mathbf{x}) , \quad (2.24)$$

where $H(t, \mathbf{x})$ is the local Hubble expansion rate due to the perturbations. Then the primordial curvature perturbation can be expressed as

$$\zeta(t, \mathbf{x}) = N(t, t_i, \mathbf{x}) - N_0(t, t_i) \equiv \delta N , \quad (2.25)$$

¹In this work we consider the case with c_s is very close to one by assuming $u \sim \mathcal{O}(\epsilon)$. In general, one may consider models with an arbitrary c_s , however, as addressed in [45, 46] where the multiple DBI inflation is investigated, the multi-field effect is suppressed in the limit of $c_s \ll 1$, which may be a possible source of large non-Gaussianity in single field models. We will leave this for a future investigation.

²From (2.10), (2.22)-(2.23), we can extract some useful relations for later convenience

$$X P_{,X} = \Sigma c_s^2 , \quad X^2 P_{,XX} = \Sigma(1 - c_s^2)/2 , \quad X^3 P_{,XXX} = 3\lambda/2 - 3\Sigma(1 - c_s^2)/4 , \quad \frac{1}{c_s^2} - 1 = 2X \frac{P_{,XX}}{P_{,X}} .$$

where $N_0(t, t_i)$ is the background (unperturbed) expansion, which is related to the background Hubble expansion rate $H_0(t)$ as

$$N_0(t, t_i) = \int_{t_i}^t dt' H_0(t') . \quad (2.26)$$

If we take t_i at the time of horizon-crossing during inflation denoted by t_* , then $N(t, t_*, \mathbf{x})$ becomes a function of the scalar fields evaluated at horizon-crossing. Then ζ can be expanded as

$$\zeta(t, \mathbf{x}) = \sum_I N_{,I}(t) \delta\phi_*^I(\mathbf{x}) + \frac{1}{2} \sum_{I,J} N_{,IJ}(t) \delta\phi_*^I(\mathbf{x}) \delta\phi_*^J(\mathbf{x}) + \cdots , \quad (2.27)$$

Note that $\zeta(t)$ is just the primordial adiabatic curvature perturbation if we choose t well after the reheating process.

After going to the momentum space, we have

$$\zeta(\mathbf{k}) = N_{,I} \delta\phi^I(\mathbf{k}) + \frac{1}{2} N_{,IJ} [\delta\phi^I * \delta\phi^J](\mathbf{k}) + \cdots . \quad (2.28)$$

The two-point and three-point functions of ζ can be expressed in terms of the two and three-point functions of the scalar fields fluctuations $\delta\phi^I$ as

$$\langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \rangle = N_{,I} N_{,J} \langle \delta\phi^I(\mathbf{k}_1) \delta\phi^J(\mathbf{k}_2) \rangle + \cdots , \quad (2.29)$$

$$\begin{aligned} \langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \zeta(\mathbf{k}_3) \rangle &= N_{,I} N_{,J} N_{,K} \langle \delta\phi^I(\mathbf{k}_1) \delta\phi^J(\mathbf{k}_2) \delta\phi^K(\mathbf{k}_3) \rangle \\ &\quad + \frac{1}{2} N_{,I} N_{,J} N_{,KL} \langle \delta\phi^I(\mathbf{k}_1) \delta\phi^J(\mathbf{k}_2) [\delta\phi^K * \delta\phi^L](\mathbf{k}_3) \rangle + \text{perms} + \cdots , \end{aligned} \quad (2.30)$$

where $*$ denotes the convolution product. In Section 3.2 we can see that the two-point functions for the scalar fields satisfy

$$\langle \delta\phi^I(\mathbf{k}_1) \delta\phi^J(\mathbf{k}_2) \rangle = (2\pi)^3 \delta^2(\mathbf{k}_1 + \mathbf{k}_2) G^{IJ} \frac{2\pi^2}{k_1^3} \Delta_\star^2 , \quad (2.31)$$

and in Section 5 we will show that the scalar three-point functions can be written in the form

$$\langle \delta\phi^I(\mathbf{k}_1) \delta\phi^J(\mathbf{k}_2) \delta\phi^K(\mathbf{k}_3) \rangle = (2\pi)^2 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \frac{4\pi^4}{\prod_i k_i^3} |\Delta_\star^2|^2 \mathcal{A}^{IJK} , \quad (2.32)$$

where Δ_\star^2 is the power spectrum of a massless scalar field in de Sitter space. The principal result of this work is the momentum-dependent function $\mathcal{A}^{IJK}(k_1, k_2, k_3)$ given in (5.15), which contains the information of the amplitude and shape of the non-Gaussianity.

In order to connect the above analysis with the observations, the non-Gaussianity measured by the three-point functions must be expressed in terms of an experimentally relevant parameter. A common choice is the non-linearity parameter f_{NL} defined as

$$\zeta = \zeta_g + \frac{3}{5} f_{\text{NL}} \zeta_g^2 , \quad (2.33)$$

which denotes the departure of ζ from a Gaussian random variable ζ_g . The power spectrum and bispectrum of ζ are defined in terms of the two and three-point functions respectively as

$$\begin{aligned} \langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \rangle &= (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2) P_\zeta(k_1) , \\ \langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \zeta(\mathbf{k}_3) \rangle &= (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\zeta(k_1, k_2, k_3) , \end{aligned} \quad (2.34)$$

then B_ζ is related with P_ζ in terms of f_{NL} as

$$B_\zeta = \frac{6}{5} f_{\text{NL}}(k_1, k_2, k_3) [P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1)] . \quad (2.35)$$

To relate f_{NL} with the three-point function of ζ , we write

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3) \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \frac{4\pi^4}{\prod_i k_i^3} |\Delta_\star^2|^2 \mathcal{A}_\zeta , \quad (2.36)$$

then f_{NL} can be written as

$$f_{\text{NL}} = \frac{5}{6} \frac{\mathcal{A}_\zeta}{\sum_i k_i^3} . \quad (2.37)$$

From (2.29) and (2.30), \mathcal{A}_ζ can also be written as

$$\mathcal{A}_\zeta = N_{,I} N_{,J} N_{,K} \mathcal{A}^{IJK} + G^{IK} G^{JL} N_{,I} N_{,J} N_{,KL} \sum_i k_i^3 . \quad (2.38)$$

From the above relations, the non-linearity parameter f_{NL} can be expressed in terms of $N_{,I}$ and the momentum-dependent function \mathcal{A}^{IJK} , up to leading orders, as

$$f_{\text{NL}} = \frac{5}{6} \frac{N_{,I} N_{,J} N_{,K} \mathcal{A}^{IJK}}{(G^{IJ} N_{,I} N_{,J})^2 \sum_i k_i^3} + \frac{5}{6} \frac{G^{IK} G^{JL} N_{,I} N_{,J} N_{,KL}}{(G^{IJ} N_{,I} N_{,J})^2} + \dots , \quad (2.39)$$

where ‘ \dots ’ denotes the remaining cross terms from (2.30) which we have neglected together with other higher-order terms. This expression was first derived in [35].

Furthermore, in order to calculate the primordial power spectrum and the non-linearity parameter, we need to know the derivatives of the number of e-folding N with respect to the scalar fields $N_{,I}$, $N_{,IJ}$, etc. It is easy to show that $dN = -d \ln H / \epsilon$, and thus

$$N_{,I} = -\sqrt{\frac{P_{,X}}{2}} \frac{\epsilon_I}{\epsilon} + \dots . \quad (2.40)$$

3. Linear Perturbations

3.1 ADM formalism and the constraint equations

In single field inflation models, we have two different gauge choices. One is the gauge where we consider the curvature scalar on uniform density hypersurfaces, defined by $\delta\phi = 0$; the other is the spatially flat gauge, where we choose the uniform curvature slicing, and the spatial part of the metric is unperturbed. The physical degrees of freedom of the perturbations are completely described by the perturbations of the metric in the first gauge, and only by the perturbations of the scalar fields in the spatially flat gauge. In single-field case, both two gauge choices are natural. However, in the case of multi-field models, the first gauge is no longer possible, and the only natural choice is the spatially flat gauge, where the physical degrees of freedom are perturbations of the scalar fields.

It is very convenient to work in the ADM metric formalism

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt) , \quad (3.1)$$

where N is the lapse function and N^i is the shift vector. The ADM formalism is convenient because the equations of motion for N and N^i are exactly the energy and momentum constraints which are quite easy to solve.

Under the ADM metric ansatz, the action becomes³

$$S = \int dt d^3x \sqrt{h} N \left(\frac{R^{(3)}}{2} + P \right) + \int dt d^3x \frac{\sqrt{h}}{2N} (E_{ij} E^{ij} - E^2) , \quad (3.2)$$

³Here and in what follows, the spatial indices i, j are raised and lowered using h_{ij} .

where $h = \det h_{ij}$ and the symmetric tensor E_{ij} is defined as

$$E_{ij} = \frac{1}{2} \left(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i \right) , \quad (3.3)$$

and $E \equiv \text{tr} E_{ij} = h^{ij} E_{ij}$. $R^{(3)}$ is the three-dimensional Ricci curvature which is computed from the metric h_{ij} . The kinetic term X now can be written as

$$X = \frac{1}{2N^2} G_{IJ} \pi^I \pi^J - \frac{G_{IJ}}{2} \partial^i \phi^I \partial_i \phi^J , \quad (3.4)$$

with

$$\pi^I = \dot{\phi}^I - N^i \partial_i \phi^I . \quad (3.5)$$

The equations of motion for N and N^i give the energy constrain and momentum constraint respectively

$$P - \frac{1}{2N^2} (E_{ij} E^{ij} - E^2 + 2P_{,X} G_{IJ} \pi^I \pi^J) = 0 , \quad (3.6)$$

$$\nabla_j \left(\frac{1}{N} (E_i^j - E \delta_i^j) \right) = \frac{P_{,X}}{N} G_{IJ} \pi^I \partial_i \phi^J . \quad (3.7)$$

In spatially flat gauge, we have $h_{ij} = a^2(t) \delta_{ij}$ and thus $R^{(3)} = 0$. The unperturbed flat FRW background corresponds to $N = 1$, $N^i = 0$. The scalar fields on the flat hypersurfaces can be decomposed into

$$\phi^I(t, \vec{x}) = \phi_0^I(t) + Q^I(t, \vec{x}) , \quad (3.8)$$

where ϕ_0^I are the spatially homogeneous background values, and Q^I are the linear perturbations. In what follows, we always drop the subscript ‘0’ on ϕ_0^I and simply identify ϕ^I as the unperturbed background fields. In order to study the scalar perturbations of the metric and the scalar fields, we may expand N and N^i as

$$\begin{aligned} N &= 1 + \alpha_1 + \alpha_2 + \dots , \\ N^i &= \partial^i \beta = \partial^i (\beta_1 + \beta_2 + \dots) , \end{aligned} \quad (3.9)$$

where α_n, β_n are of order $\mathcal{O}(Q^n)$. One can plug the above power expansions into the constrain equations of N and N^i (3.6)-(3.7) to determine α_n and β_n . To the first-order of Q^I the solutions are [25]

$$\alpha_1 = \frac{P_{,X}}{2H} \dot{\phi}_I Q^I , \quad (3.10)$$

and

$$\partial^2 \beta_1 = \frac{a^2}{2H} \left[-\frac{P_{,X}}{c_s^2} \dot{\phi}_I \dot{Q}^I - 2X P_{,XI} Q^I + P_{,I} Q^I + \frac{P_{,X}}{H} \left(\frac{X P_{,X}}{c_s^2} - 3H^2 \right) \dot{\phi}_I Q^I \right] , \quad (3.11)$$

where $\partial^2 = \delta^{ij} \partial_i \partial_j$. Fortunately, it turns out that in order to expand the effective action to order $\mathcal{O}(Q^3)$, in the ADM formalism we do not need to compute N and N^i to order $\mathcal{O}(Q^3)$, since they must be multiplied by $\partial L / \partial N$ or $\partial L / \partial N^i$ which vanish due to the constraint equations. Also in the present case, terms of order $\mathcal{O}(Q^2)$ in N and N^i drop out of the third-order effective action, and thus (3.10)-(3.11) are sufficient for our purpose. Furthermore, it is easy to see that α_1 and β_1 are both of order $\mathcal{O}(\epsilon^I)$.

3.2 The second-order theory

In the Appendix A, the general form of the expansion of the action to the cubic-order of Q^I has been developed. From (A.11), the second-order action can be written as⁴ [25]

$$S^{(2)} = \frac{1}{2} \int dt d^3x a^3 \left[\left(P_{,X} G_{IJ} + P_{,XX} \dot{\phi}_I \dot{\phi}_J \right) \dot{Q}^I \dot{Q}^J + 2 \dot{\phi}_I P_{,XJ} \dot{Q}^I Q^J - P_{,X} G_{IJ} \partial^i Q^I \partial_i Q^J - \mathcal{M}_{IJ} Q^I Q^J \right], \quad (3.12)$$

where \mathcal{M}_{IJ} is the effective mass-matrix

$$\mathcal{M}_{IJ} = -P_{,IJ} + \frac{2XP_{,X}}{H} \dot{\phi}_I P_{,XJ} + \frac{XP_{,X}^3}{2H^2} \left(1 - \frac{1}{c_s^2} \right) \dot{\phi}_I \dot{\phi}_J - \frac{1}{a^3} \frac{d}{dt} \left[\frac{a^3}{2H} P_{,X}^2 \left(1 + \frac{1}{c_s^2} \right) \dot{\phi}_I \dot{\phi}_J \right]. \quad (3.13)$$

(3.12) is exact and is valid for arbitrary scalar fields dynamics.

In general, one may decompose the multi-field perturbations into an adiabatic mode and $\mathcal{N} - 1$ entropy modes, since the background trajectory specifies a special inflaton direction. Here in this work, under the assumption that c_s is close to unity, we may take a different method, to analyze all these perturbations in a unified and more simpler formalism. To solve the second-order action (3.12), we define a symmetric matrix α_{IJ} which satisfies

$$\alpha_{IK} \alpha_{KJ} = a^2 P_{,X} \mathcal{A}_{IJ}, \quad (3.14)$$

with

$$\mathcal{A}_{IJ} \equiv G_{IJ} + \frac{P_{,XX}}{a^2 P_{,X}} \phi'_I \phi'_J = G_{IJ} + \frac{u}{\epsilon} \epsilon_{IJ}, \quad (3.15)$$

where a prime denotes the derivative with respect to the conformal time η defined by $d\eta = dt/a$. Here \mathcal{A}_{IJ} denotes the deviation from the standard canonical action, which reduces to $\mathcal{A}_{IJ} = G_{IJ}$ when considering the standard kinetic terms, where $u = 0$. Then we define the new Mukhanov-type variables

$$u^I = \alpha^{IJ} Q^J, \quad Q^I = (\alpha^{-1})^{IJ} u^J, \quad (3.16)$$

or in matrix form $u = \alpha Q$ and $Q = \alpha^{-1} u$. This treatment is an analogue of that in the single-field case. For standard single-field inflation models with a canonical kinetic term, α_{IJ} reduces to a , and $u_I = \alpha_{IJ} Q_J$ reduces to the usual rescale relation $u = a\delta\phi$, and thus the matrix α_{IJ} can be viewed as the generalization of the parameter a in single-field case, where we may rescale the field as $u = a\delta\phi$.

In terms of these new variables u^I , after changing into conformal time η , the second-order action (3.12) can be rewritten in a matrix form as

$$S^{(2)} = \int d\eta d^3x \left[u'^T u' + u'^T \left(-2\alpha' \alpha^{-1} + 2a^2 \alpha^{-1} \mathcal{B} \alpha^{-1} \right) u - a^2 \mathcal{A}^{-1} \partial^i u^T \partial_i u + u^T \left(-2a^2 \alpha^{-1} \alpha' \alpha^{-1} \mathcal{B} \alpha^{-1} - a^4 \alpha^{-1} \mathcal{M} \alpha^{-1} + \alpha^{-1} \alpha' \alpha' \alpha^{-1} \right) u \right], \quad (3.17)$$

where we have defined $\mathcal{B}_{IJ} = \phi'_I P_{,XJ}$ for short. In deriving (3.17) we used the fact that $\alpha^{-1} \alpha^{-1} = \mathcal{A}^{-1} / a^2 P_{,X}$. The equations of motion for the scalars can be derived from (3.17) and written in a compact form

$$u'' + \left[\alpha^{-1} \alpha' - \alpha' \alpha^{-1} + a^2 \alpha^{-1} (\mathcal{B} - \mathcal{B}^T) \alpha^{-1} \right] u' + \left[\mathcal{A}^{-1} k^2 + a^2 \alpha^{-1} (a^2 \mathcal{M} + 2\mathcal{H} \mathcal{B} + \mathcal{B}' - (\mathcal{B} - \mathcal{B}^T) \alpha^{-1} \alpha') \alpha^{-1} - \alpha'' \alpha^{-1} \right] u = 0, \quad (3.18)$$

where $\mathcal{H} = a'/a$. Note that (3.18) is exact and no approximation is made. For standard single field models, (3.18) reduces to the well-known result $u'' + (k^2 + a^2 m^2 - a''/a)u = 0$ as expected. From (3.15)

⁴This expression should be compared, for example, with eq. (44) of [36], to which it reduces in the multi-field models with canonical kinetic terms where $P = X - V$.

it can be seen easily that in the single-field case, the matrix \mathcal{A}_{IJ} reduces to a pure number $\mathcal{A} = 1/c_s^2$, and thus the first term in the second line, which is proportional to the wave number k , reduces to $c_s^2 k^2$ as expected.

We would like to make some comments here. From (3.15) we can see that the unperturbed background fields velocities ϕ'_I , represent a special direction in field space when considering perturbations of the fields. This is just the so-called adiabatic direction which has been introduced in [52] for multi-field inflation models with canonical kinetic terms. The decomposition into adiabatic and entropy modes is equivalent to a ‘local’ rotation in the space of field perturbations,

$$\tilde{Q}^n = e_I^n Q^I, \quad Q^I = e_n^I \tilde{Q}^n, \quad (3.19)$$

the rotation matrix e_I^n is just the projection of a new set of basis $\{e^n\}$ on $\{\phi_I\}$. The first vector is specified as

$$e_I^1 = \frac{\dot{\phi}_I}{\sqrt{2X}}, \quad (3.20)$$

which is just the unit local adiabatic vector. Note that the rotation e_I^n is defined locally, and depends on the background trajectory. The new field \tilde{Q}^1 is just the adiabatic mode while other modes \tilde{Q}^n ($n = 2, \dots, \mathcal{N}$) are entropy modes. After the local rotation, i.e the decomposition into adiabatic and entropy modes, it is easy to show that $\mathcal{A}_{IJ} = G_{IJ} + (1/c_s^2 - 1) e_I^1 e_J^1$, and

$$\mathcal{A}_{IJ} \dot{Q}^I \dot{Q}^J = \frac{1}{c_s^2} \left(\dot{\tilde{Q}}^1 + Z_p^1 \tilde{Q}^p \right) \left(\dot{\tilde{Q}}^1 + Z_q^1 \tilde{Q}^q \right) + \sum_{m \neq 1} \left(\dot{\tilde{Q}}^m + Z_p^m \tilde{Q}^p \right) \left(\dot{\tilde{Q}}^m + Z_q^m \tilde{Q}^q \right), \quad (3.21)$$

where we denote $Z_n^m = e_I^m \dot{e}_n^I$ for short. This decomposition clearly shows that the adiabatic component of the perturbations \tilde{Q}^1 obeys a wave equation where the propagation speed is the sound speed c_s , while the entropy modes of the perturbations \tilde{Q}^n ($n = 2, \dots, \mathcal{N}$) propagate with the speed of light $c = 1$. This property was first pointed out in the studying of two-field DBI-inflation [45, 46], but here we see that it turns out to be a generic feature for general multiple field inflation models.

The decomposition into adiabatic and entropy modes is a powerful tool to analyse the equations (3.18). In general, however, (3.18) is a set of coupled equations and rather complicated to solve, even in the case of canonical kinetic term. The idea is to take slow-roll approximations as in the standard slow-roll inflation models. This is the standard approximation in estimating the amplitude of the perturbation power spectra, i.e. the two-point functions. For our purpose, it also supplies a unified treatment with all these perturbations Q^I , rather than decomposing them into adiabatic and entropy modes.

In single-field models, the mass term can be expressed as a combination of slow-roll parameters, and thus be neglected to leading-order in slow-roll approximation. Similarly, the mass-matrix defined in (3.13) can also be written in terms of slow-varying parameters as

$$\begin{aligned} \frac{1}{H^2} \mathcal{M}_{IJ} &= -3P_{,X} \left(1 + \frac{1}{c_s^2} \right) \epsilon_{IJ} + 3\tilde{\eta}_{IJ} \\ &\quad - \frac{12}{\sqrt{P_{,X}}} \epsilon \epsilon_I \left(\frac{P_{,X}}{3c_s^2} \tilde{\epsilon}_I + \tilde{\epsilon}_{I,X} \right) - \frac{P_{,X}}{H} \left(1 + \frac{1}{c_s^2} \right) \epsilon_{IJ} + P_{,X} \left(1 + \frac{1}{c_s^2} \right) \epsilon (\epsilon_{IJ} - \eta_{IJ}) + \frac{2P_{,X}}{c_s^2} s \epsilon_{IJ} \\ &= -6P_{,X} \epsilon_{IJ} + 3\tilde{\eta}_{IJ} + \mathcal{O}(\epsilon^2). \end{aligned} \quad (3.22)$$

We also note that $\mathcal{B}_{IJ} = \phi'_I P_{,XJ} \sim \mathcal{O}(\epsilon)$. Since we assume that c_s^2 departs from unity by a quantity that is first-order in slow-roll, i.e. $(1/c_s^2 - 1) \sim \mathcal{O}(\epsilon)$, \mathcal{A}_{IJ} can be written as

$$\mathcal{A}_{IJ} = G_{IJ} + \mathcal{O}(\epsilon), \quad (3.23)$$

and thus $(\mathcal{A}^{-1})_{IJ} = G_{IJ} - \mathcal{O}(\epsilon)$. Furthermore, from (3.14) it is easy to show that the last term in (3.18) can be written as

$$\alpha'' \alpha^{-1} = \frac{a''}{a} (1 + \mathcal{O}(\epsilon)) . \quad (3.24)$$

Bring all these considerations together, the equations of motion to the lowest-order of slow-varying parameters reduce to a very simple form which are just \mathcal{N} decoupled de Sitter-Mukhanov equations

$$u^{I''} + \left(k^2 - \frac{a''}{a} \right) u^I = 0 , \quad (3.25)$$

where $a''/a = 2/\eta^2 + \mathcal{O}(\epsilon)$.

The solutions to (3.25) are standard, we then find the Q^I two-point functions as

$$\langle Q^I(\eta_1, \mathbf{x}_1) Q^J(\eta_2, \mathbf{x}_2) \rangle = G^{IJ} G_\star(\eta_1, \mathbf{x}_1; \eta_2, \mathbf{x}_2) , \quad (3.26)$$

while G_\star represents

$$G_\star(\eta_1, \eta_2, \mathbf{k}) = \frac{H^2}{2k^3 P_{,X}} \times \begin{cases} (1 + ik\eta_1)(1 - ik\eta_2)e^{-ik(\eta_1 - \eta_2)} , & \eta_1 > \eta_2 \\ (1 - ik\eta_1)(1 + ik\eta_2)e^{+ik(\eta_1 - \eta_2)} , & \eta_1 < \eta_2 \end{cases} , \quad (3.27)$$

where we have chosen boundary conditions so that G_\star behaves like flat space propagator at very early times, when the perturbation modes are deep inside the horizon. This corresponds to the Bunch-Davies vacuum [53]. The power spectra for the scalar fields on large scales can be read easily

$$\langle Q^I(\mathbf{k}_1) Q^J(\mathbf{k}_2) \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2) G^{IJ} \frac{H^2}{2k_1^3 P_{,X}} , \quad (3.28)$$

and therefore the dimensionless power spectra are⁵

$$\Delta_\star^2 = \frac{H^2}{4\pi^2 P_{,X}} . \quad (3.29)$$

4. Non-linear Perturbations

4.1 General form of the third-order action

In this section we turn to the central calculation of this work, the third-order piece of the coupled action (3.2). The general form of the expansion of the action to the cubic-order of scalar fields perturbations Q^I is developed in Appendix A

$$S^{(3)} = \int dt d^3x a^3 \left(P^{(3)} + \alpha_1 P^{(2)} - \alpha_1 \Pi^{(2)} + \alpha_1^2 \Pi^{(1)} - \alpha_1^3 \Pi^{(0)} \right) , \quad (4.1)$$

⁵This result coincides with (97) and (105) of [25] in the $c_s \rightarrow 1$ limit as expected, while in [25] the adiabatic and entropy spectra are obtained respectively with an arbitrary c_s .

where α_1 is given by (3.10) and $P^{(n)}, \Pi^{(n)}$ can be found in Appendix A. After a straightforward but rather tedious calculation, we find

$$\begin{aligned}
S^{(3)} = \int dt d^3 x a^3 \left[\left(-\frac{P_{,X}^2}{4Hc_s^2} G_{IJ} \dot{\phi}_K - \frac{3\lambda P_{,X}^3}{4H\Sigma^2 c_s^4} \dot{\phi}_I \dot{\phi}_J \dot{\phi}_K + \frac{1}{2} G_{IJ} P_{,XK} + \frac{1}{2} \dot{\phi}_I \dot{\phi}_J P_{,XXK} \right) \dot{Q}^I \dot{Q}^J \dot{Q}^K \right. \\
+ \left(\frac{3\lambda P_{,X}^3}{4H\Sigma^2 c_s^2} \dot{\phi}_I \dot{\phi}_J \dot{\phi}_K - \frac{P_{,X}}{2H} \dot{\phi}_I \dot{\phi}_J P_{,XK} - \frac{\Sigma c_s^2}{H} \dot{\phi}_I \dot{\phi}_J P_{,XXK} + \frac{1}{2} \dot{\phi}_I P_{,XJK} \right) \dot{Q}^I \dot{Q}^J \dot{Q}^K \\
+ \left(-\frac{P_{,X} \Sigma c_s^2}{4H^2} \dot{\phi}_I \dot{\phi}_J P_{,XK} - \frac{P_{,X}^3}{8H^3} (2\lambda - \Sigma + 3H^2) \dot{\phi}_I \dot{\phi}_J \dot{\phi}_K + \frac{P_{,X}^2}{4H^2} \dot{\phi}_I \dot{\phi}_J P_{,K} + \frac{P_{,X}}{4H} P_{,IJ} \dot{\phi}_K \right. \\
\left. + \frac{\Sigma^2 c_s^4}{2H^2} \dot{\phi}_I \dot{\phi}_J P_{,XXK} - \frac{\Sigma c_s^2}{2H} \dot{\phi}_I P_{,XJK} + \frac{1}{6} P_{,IJK} \right) \dot{Q}^I \dot{Q}^J \dot{Q}^K \\
+ \left(\frac{1}{2} P_{,XX} G_{IJ} \dot{\phi}_K + \frac{1}{6} P_{,XXX} \dot{\phi}_I \dot{\phi}_J \dot{\phi}_K \right) \dot{Q}^I \dot{Q}^J \dot{Q}^K - \frac{P_{,X}^2}{4H} G_{IJ} \dot{\phi}_K \partial^i Q^I \partial_i Q^J \dot{Q}^K \\
- P_{,X} \dot{Q}_I \partial^i \beta_1 \partial_i Q^I + \frac{P_{,X}^2}{2H} \dot{\phi}_I \dot{\phi}_J Q^I \partial^i \beta_1 \partial_i Q^J - P_{,XX} \dot{\phi}_I \dot{\phi}_J \dot{Q}^I \partial^i \beta_1 \partial_i Q^J \\
- \frac{P_{,XX}}{2} G_{IJ} \dot{\phi}_K \partial^i Q^I \partial_i Q^J \dot{Q}^K + \frac{P_{,XX} \Sigma c_s^2}{H} \dot{\phi}_I \dot{\phi}_J Q^I \partial^i \beta_1 \partial_i Q^J + \frac{P_{,XX} \Sigma c_s^2}{2H} G_{IJ} \dot{\phi}_K \partial^i Q^I \partial_i Q^J \dot{Q}^K \\
\left. - P_{,XI} \dot{\phi}_J Q^I \partial^i \beta_1 \partial_i Q^J - \frac{1}{2} G_{IJ} P_{,XK} \partial^i Q^I \partial_i Q^J \dot{Q}^K - \frac{P_{,X}}{4H} \dot{\phi}_I Q^I (\partial^{ij} \beta_1 \partial_{ij} \beta_1 - (\partial^i \partial_i \beta_1)^2) \right] . \tag{4.2}
\end{aligned}$$

No approximation of small slow-varying parameters has been made in deriving (4.2), and thus it is exact.

4.2 Slow-varying limit

In order to proceed, we restrict (4.2) to the leading-order of slow-varying parameters. This is because of not only the complexity of the full cubic-order action (4.2), but also the observational constraints. Therefore the third-order action can be written in a much simpler form⁶

$$\begin{aligned}
S^{(3)} = \int dt d^3 x a^3 \left[\left(\frac{1}{2} P_{,XX} G_{IJ} \dot{\phi}_K + \frac{1}{6} P_{,XXX} \dot{\phi}_I \dot{\phi}_J \dot{\phi}_K \right) \dot{Q}^I \dot{Q}^J \dot{Q}^K \right. \\
+ \left(\frac{1}{2} G_{IJ} P_{,XK} - \frac{P_{,X}^2}{4H} G_{IJ} \dot{\phi}_K \right) \dot{Q}^I \dot{Q}^J \dot{Q}^K - \frac{P_{,XX}}{2} G_{IJ} \dot{\phi}_K \partial^i Q^I \partial_i Q^J \dot{Q}^K \\
\left. - \left(\frac{P_{,X}^2}{4H} G_{IJ} \dot{\phi}_K + \frac{1}{2} G_{IJ} P_{,XK} \right) \partial^i Q^I \partial_i Q^J \dot{Q}^K - P_{,X} \dot{Q}_I \partial^i \beta_1 \partial_i Q^I \right] . \tag{4.3}
\end{aligned}$$

β_1 is defined in (3.11) and is also given to the leading-order by

$$\partial^2 \beta_1 = -\frac{a^2 P_{,X}}{2H} \dot{\phi}_I \dot{Q}^I . \tag{4.4}$$

Now it proves most convenient to take integral by parts to eliminate the last term containing $\partial_i \beta_1$ in (4.3). It is easy to show that

$$\begin{aligned}
& - \int dt d^3 x a^3 P_{,X} \dot{Q}_I \partial^i \beta_1 \partial_i Q^I \\
& = \int dt d^3 x \left(-\frac{a^3 H}{2} P_{,X} \partial^i Q_I \partial_i Q^I \beta_1 - \frac{a^3}{2} P_{,X} \partial^i Q_I \partial_i Q^I \dot{\beta}_1 + a P_{,X} \dot{Q}_I \partial^2 Q^I \beta_1 - \frac{a^3}{2} P_{,X} \partial^i Q_I \partial_i Q^I \beta_1 \right) \\
& \approx \int dt d^3 x a^3 \left(-\frac{H P_{,X}}{2} \partial^i Q_I \partial_i Q^I \beta_1 - \frac{P_{,X}}{2} \partial^i Q_I \partial_i Q^I \dot{\beta}_1 + P_{,X} \dot{Q}_I \partial^i \partial_i Q^I \beta_1 \right) , \tag{4.5}
\end{aligned}$$

⁶This reduces to, for example, eq. (53) of [36] where a canonical kinetic term was considered, as expected.

where we have used the fact that $\dot{P}_{,X}/(HP_{,X})$ is order $\mathcal{O}(\epsilon)$, and we have neglected the last term in the second line in (4.5). Now (4.4) can be used to evaluate $\dot{\beta}_1$ in the last line in (4.5). Taking time derivative to both sides in (4.4) and keeping only the leading-order terms we have

$$\partial^2 \dot{\beta}_1 = -a^2 P_{,X} \dot{\phi}_I \dot{Q}^I - \frac{a^2 P_{,X}}{2H} \dot{\phi}_I \ddot{Q}^I. \quad (4.6)$$

As emphasized in [36] that the above equation cannot be inserted directly into the action (4.3) since it contains \ddot{Q}^I and therefore will change the order of the equations of motion for Q^I . However, it is very convenient to make use of the equations of motion derived from the second-order theory (3.12) to eliminate \ddot{Q}^I in (4.6). To the lowest-order of slow-varying parameters, the second-order action (3.12) reduces to the form

$$S^{(2)} = \frac{1}{2} \int dt d^3x a^3 P_{,X} G_{IJ} \left(\dot{Q}^I \dot{Q}^J - \partial^i Q^I \partial_i Q^J + \mathcal{O}(\epsilon) \right), \quad (4.7)$$

and gives

$$\left. \frac{\delta L}{\delta Q^I} \right|_1 = a^3 P_{,X} \left(-3H \dot{Q}_I - \ddot{Q}_I + \partial^i \partial_i Q_I \right) + \mathcal{O}(\epsilon). \quad (4.8)$$

This vanishes when the perturbations Q^I solve the free Gaussian theory, but $(\delta L/\delta Q^I)|_1$ will be non-zero when considering the full theory and of course the third-order interacting theory. Solving \ddot{Q}^I from (4.8) and inserting them into (4.6), it follows that

$$\partial^2 \dot{\beta}_1 = \frac{a^2 P_{,X}}{2} \dot{\phi}_I \dot{Q}^I - \frac{P_{,X}}{2H} \dot{\phi}_I \partial^2 Q^I + \frac{1}{2Ha} \dot{\phi}^I \left. \frac{\delta L}{\delta Q^I} \right|_1 + \mathcal{O}(\epsilon). \quad (4.9)$$

Substituting (4.9) into (4.5) and performing a lot of integrals by parts, we finally get the equivalent third-order action

$$\begin{aligned} S^{(3)} = \int dt d^3x \left\{ \frac{a^3}{2} \left(P_{,XX} G_{IJ} \dot{\phi}_K + \frac{1}{3} P_{,XXX} \dot{\phi}_I \dot{\phi}_J \dot{\phi}_K \right) \dot{Q}^I \dot{Q}^J \dot{Q}^K \right. \\ + a^3 \left(\frac{1}{2} G_{IJ} P_{,XK} - \frac{P_{,X}^2}{4H} G_{IJ} \dot{\phi}_K \right) \dot{Q}^I \dot{Q}^J \dot{Q}^K \\ - \frac{a^3 P_{,XX}}{2} G_{IJ} \dot{\phi}_K \partial^i Q^I \partial_i Q^J \dot{Q}^K - \frac{a^3}{2} G_{IJ} P_{,XK} \partial^i Q^I \partial_i Q^J \dot{Q}^K \\ \left. - \frac{a^3 P_{,X}^2}{2H} G_{IJ} \dot{\phi}_K \dot{Q}^I \partial^2 Q^J \partial^{-2} \dot{Q}^K + \left. \frac{\delta L}{\delta Q^I} \right|_1 F^I(Q) \right\}, \end{aligned} \quad (4.10)$$

with

$$F^I(Q) = \frac{P_{,X}}{4H} \dot{\phi}^I \left(\partial^{-2} (Q_J \partial^2 Q^J) - \frac{1}{2} Q_J Q^J \right). \quad (4.11)$$

The last term $\left. \frac{\delta L}{\delta Q^I} \right|_1 F^I(Q)$ in (4.10) which is proportional to $(\delta L/\delta Q^I)|_1$ can be absorbed by a fields redefinition of Q^I into new fields \mathcal{Q}^I , as in the single-field case. It can be shown easily that the appropriate fields redefinition is

$$Q^I = \mathcal{Q}^I - F^I(\mathcal{Q}) = \mathcal{Q}^I + \frac{P_{,X}}{8H} \dot{\phi}^I \mathcal{Q}_J \mathcal{Q}^J - \frac{P_{,X}}{4H} \dot{\phi}^I \partial^{-2} (\mathcal{Q}_J \partial^2 \mathcal{Q}^J). \quad (4.12)$$

Such a fields redefinition where F^I are quadratic in Q^I , has no effect on any of the $\mathcal{O}(Q^3)$ terms in the third-order action (4.10), and thus we may simply replace Q^I with \mathcal{Q}^I there. On the other hand, the fields redefinition indeed modifies the quadratic part of the action, i.e. the Gaussian action (3.12), which transforms as

$$S^{(2)}[Q] \mapsto S^{(2)}[\mathcal{Q}] - \int dt d^3x \left. \frac{\delta L}{\delta Q^I} \right|_1 F^I(\mathcal{Q}), \quad (4.13)$$

the second term here cancels the last term in (4.10) exactly, which is proportional to the first-order equations of motion $\left. \frac{\delta L}{\delta Q^I} \right|_1$.

5. Calculating the Three Point Function

In this section, we proceed to calculate the scalar fields three-point functions $\langle Q^I(\mathbf{k}_1)Q^J(\mathbf{k}_2)Q^K(\mathbf{k}_3) \rangle$, with the third-order perturbative action (4.10) derived in the above section. The calculation of the three-point functions is standard, and thus we simply collect the final results here.

1. Contribution from $\dot{Q}^I\dot{Q}^J\dot{Q}^K$ interaction.

In conformal time η , this interaction can be written as

$$\int d\eta d^3x \frac{a}{H} f_{(1)IJK} Q'^I Q'^J Q'^K, \quad (5.1)$$

with dimensionless coefficient

$$\begin{aligned} f_{(1)IJK} &\equiv \frac{H}{2} \left(P_{,XX} G_{IJ} \dot{\phi}_K + \frac{1}{3} P_{,XXX} \dot{\phi}_I \dot{\phi}_J \dot{\phi}_K \right) \\ &= \left(\frac{P_{,X}}{2} \right)^{3/2} \left[G_{IJ} + \left(\frac{2\lambda}{H^2 \epsilon u} - 1 \right) \frac{\epsilon_{IJ}}{\epsilon} \right] \frac{u}{\epsilon} \epsilon_K, \end{aligned} \quad (5.2)$$

where we have expressed $f_{(1)IJK}$ in terms of slow-varying parameters defined in Section 2.2. After a standard calculation we find the contribution from this term as

$$\begin{aligned} &i(2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) f_{(1)IJK} \frac{H^6}{P_{,X}^3 \prod_i (2k_i^3)} \int_{-\infty}^0 d\eta \frac{1}{-H^2 \eta} k_1^2 k_2^2 k_3^2 \eta^3 e^{+iK\eta} + \text{perms} + \text{c.c.} \\ &= (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) f_{(1)*}^{IJK} \frac{H_*^4}{P_{,X*}^3 \prod_i (2k_i^3)} \frac{4k_1^2 k_2^2 k_3^2}{K^3} + \text{perms}, \end{aligned} \quad (5.3)$$

where $K = k_1 + k_2 + k_3$, and an asterisk ‘*’ denotes that the corresponding quantities are evaluated at horizon crossing $k = aH$. Here “permutation” means total 6 ways of simultaneously rearranging the indices I, J and K and momenta k_1, k_2 and k_3 (i.e. the index ‘I’ is always tied to k_1 , and so on).

2. Contribution from $\dot{Q}^I\dot{Q}^JQ^K$ interaction.

$$(2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) f_{(2)*}^{IJK} \frac{2H_*^4}{P_{,X*}^3 \prod_i (2k_i^3)} \left(\frac{k_1^2 k_2^2}{K} + \frac{k_1^2 k_2^2 k_3}{K^2} \right) + \text{perms}, \quad (5.4)$$

with

$$\begin{aligned} f_{(2)IJK} &= \frac{1}{2} G_{IJ} P_{,XK} - \frac{P_{,X}^2}{4H_c^2} G_{IJ} \dot{\phi}_K \\ &= - \left(\frac{P_{,X}}{2} \right)^{3/2} G_{IJ} \left(\frac{3u}{2\epsilon} \tilde{\epsilon}_K + 2\tilde{\epsilon}_K + \frac{6H^2}{P_{,X}} \tilde{\epsilon}_{K,X} + \epsilon_K \right). \end{aligned} \quad (5.5)$$

3. Contribution from $\partial^i Q^I \partial_i Q^J \dot{Q}^K$ interaction.

$$(2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) f_{(3)*}^{IJK} \frac{2H_*^4}{P_{,X*}^3 \prod_i (2k_i^3)} (\mathbf{k}_1 \cdot \mathbf{k}_2) k_3^2 \left(\frac{k_1 + k_2}{K^2} + \frac{2k_1 k_2}{K^3} + \frac{1}{K} \right) + \text{perms}, \quad (5.6)$$

with

$$f_{(3)IJK} = -\frac{H}{2} P_{,XX} G_{IJ} \dot{\phi}_K = - \left(\frac{P_{,X}}{2} \right)^{3/2} \frac{u}{\epsilon} G_{IJ} \epsilon_K. \quad (5.7)$$

4. Contribution from $\partial^i Q^I \partial_i Q^J Q^K$ interaction.

$$(2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) f_{(4)*}^{IJK} \frac{2H_*^4}{P_{,X*}^3 \prod_i (2k_i^3)} \mathbf{k}_1 \cdot \mathbf{k}_2 \left(-K + \frac{\sum_{i>j} k_i k_j}{K} + \frac{k_1 k_2 k_3}{K^2} \right) + \text{perms}, \quad (5.8)$$

with

$$f_{(4)IJK} = -\frac{1}{2}G_{IJ}P_{,XK} = \left(\frac{P_{,X}}{2}\right)^{3/2} G_{IJ} \left(\frac{3u}{2\epsilon}\tilde{\epsilon}_K + 2\tilde{\epsilon}_K + \frac{6H^2}{P_{,X}}\tilde{\epsilon}_{K,X}\right). \quad (5.9)$$

5. Contribution from $\dot{Q}^I \partial^2 Q^J \partial^{-2} \dot{Q}^K$ interaction.

$$(2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) f_{(5)*}^{IJK} \frac{2H_*^4}{P_{,X*}^3 \prod_i (2k_i^3)} \left(\frac{k_1^2 k_2^2}{K} + \frac{k_1^2 k_3^2}{K^2}\right) + \text{perms}, \quad (5.10)$$

with

$$f_{(5)IJK} = -\frac{P_{,X}^2}{2H} G_{IJ} \dot{\phi}_K = -2 \left(\frac{P_{,X}}{2}\right)^{3/2} G_{IJ} \epsilon_K. \quad (5.11)$$

6. Contribution from the fields redefinition $Q^I \mapsto \mathcal{Q}^I - F^I(\mathcal{Q})$.

Redefinition $Q^I \mapsto \mathcal{Q}^I + \frac{P_{,X}}{8H} \dot{\phi}^I \mathcal{Q}_J \mathcal{Q}^J$ gives

$$(2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \frac{H_*^4}{P_{,X*}^3 \prod_i (2k_i^3)} \left(\frac{P_{,X*}}{2}\right)^{3/2} \epsilon^I G^{JK} k_1^3 + \text{perms}, \quad (5.12)$$

and redefinition $Q^I \mapsto \mathcal{Q}^I - \frac{P_{,X}}{4H} \dot{\phi}^I \partial^{-2} (\mathcal{Q}_J \partial^2 \mathcal{Q}^J)$ gives

$$-(2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \frac{H_*^4}{P_{,X*}^3 \prod_i (2k_i^3)} \left(\frac{P_{,X*}}{2}\right)^{3/2} \epsilon^I G^{JK} k_1 k_3^2 + \text{perms}. \quad (5.13)$$

Bring all these contributions together, after some simplifications, we find an expression for the scalar fields perturbations three-point correlation functions

$$\langle Q^I(\mathbf{k}_1) Q^J(\mathbf{k}_2) Q^K(\mathbf{k}_3) \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \frac{H_*^4}{(2P_{,X})^{3/2} \prod_i (2k_i^2)} \tilde{\mathcal{A}}^{IJK}, \quad (5.14)$$

with

$$\begin{aligned} \tilde{\mathcal{A}}^{IJK} = & G^{IJ} \epsilon^K \frac{u}{\epsilon} \left[\frac{4k_1^2 k_2^2 k_3^2}{K^3} - 2(\mathbf{k}_1 \cdot \mathbf{k}_2) k_3 \left(\frac{1}{K} + \frac{k_1 + k_2}{K^2} \frac{2k_1 k_2}{K^3} \right) \right] \\ & - G^{IJ} \epsilon^K \left[6 \frac{k_1^2 k_2^2}{K} + 2 \frac{k_1^2 k_2^2 (k_3 + 2k_2)}{K^2} + k_3 k_2^2 - k_3^3 \right] \\ & + G^{IJ} \left(3 \frac{u}{\epsilon} \tilde{\epsilon}^K + 4\tilde{\epsilon}^K + \tilde{\epsilon}_{,X}^K \frac{12H^2}{P_{,X}} \right) \left[-\frac{k_1^2 k_2^2}{K} - \frac{k_1^2 k_2^2 k_3}{K^2} + (\mathbf{k}_1 \cdot \mathbf{k}_2) \left(-K + \frac{\sum_{i>j} k_i k_j}{K} + \frac{k_1 k_2 k_3}{K^2} \right) \right] \\ & + \frac{\epsilon^{IJ}}{\epsilon} \epsilon^K \left(\frac{2\lambda}{H^2 \epsilon^2} - \frac{u}{\epsilon} \right) \frac{4k_1^2 k_2^2 k_3^2}{K^3} + \text{perms}, \end{aligned} \quad (5.15)$$

and \mathcal{A}_{IJK} defined in (2.32) is related to the above by

$$\mathcal{A}_{IJK} = \frac{1}{4} \sqrt{\frac{P_{,X}}{2}} \tilde{\mathcal{A}}_{IJK}. \quad (5.16)$$

Here the Hubble parameter H , effective speed of sound c_s , slow-varying parameters ϵ^I etc. and λ are all evaluated at the time of horizon-crossing $k \approx aH$. In general, with an arbitrary c_s , the adiabatic mode exits the horizon at $c_s k = aH$, as addressed by many authors [45, 25]. In this paper we assume that $u = 1/c_s^2 - 1$ is of $\mathcal{O}(\epsilon)$.

The above result (5.14)-(5.16) reduces to that of [36], where the three-point functions of multi-field models with canonical kinetic term $P = X - V$ have been investigated. Our result shows the dependence of non-Gaussianity on these slow-varying parameters. As argued in [27], where a very general single field model was considered, our results shows that the final non-Gaussianity is

proportional to these small slow-varying parameters, except a parameter λ . However, from the last line in (5.15) it is easy to see that the large non-Gaussianities would arise in models with $-\frac{\lambda}{H^2\epsilon} \gg 1$ during inflation. Note that the standard choice of kinetic term corresponds to the case $\lambda = 0$. Models with large u , i.e. $c_s \ll 1$ may be another source of large non-Gaussianity, as argued in [27], and it will be of great interest, we would like to leave this for a future investigation.

6. Conclusion

In this paper, we have calculated the three-point function of curvature perturbation at the horizon-crossing, which arises from a general multiple-field inflation model with the action (2.1). We use δN formalism to relate the curvature perturbation ζ and the multiple scalar fields fluctuations. The result is given by (2.39) and (5.14)-(5.16), where the momentum-dependent amplitude \mathcal{A}^{IJK} is given by (5.15)-(5.16). This result includes a large class of inflationary models, and can be viewed as a multiple-field generalization of previous results considering k-inflation [16], ghost condensation [19] and DBI-inflation [45, 46, 47]. In the case of standard canonical kinetic term, our result reduces to the previous known results [36, 73, 28], where the method of Lagrangian formalism or the fields equations recently developed is used. Our formalism would be helpful to analyze the dependence of non-Gaussianities on the structure of the inflation models, and also be useful to study the non-Gaussianities in multi-field inflationary models which will be constructed in the future.

In the presence of many light scalar fields coupled to Einstein gravity with a non-canonical kinetic term, we define some slow-varying parameters in order to control the theory and to find the solutions perturbatively in these small parameters. These parameters can be seen as a generalization of the standard slow-roll parameters. However, in this paper we do not ascertain the conditions under which a particular $P(X, \phi^I)$ would admit such a slow-varying limit. If this limit breaks down, the theory would become rather complicated and the calculation would be less clear. Moreover, as addressed in [26, 27] and etc., in the presence of a non-canonical kinetic term, the smallness of these slow-varying parameters do not imply that the inflation itself is slow-rolling.

In multi-field inflation models, this microphysically-originated non-Gaussianities produced at the horizon-crossing provide the initial conditions for the superhorizon evolution of the non-Gaussianities afterwards [35]. Even though the “initial” non-Gaussianity is small, after the superhorizon evolution, the final “primordial” non-Gaussianity would be significantly large, as in the curvaton scenarios [54, 55, 56, 57, 58, 59, 72, 61].

In this work, we studied the non-Gaussianities which are generated due to the non-linear relations between curvature and inflaton perturbations. In general, primordial density perturbations can be generated not only from the inflaton field(s) during inflation. There are also other mechanisms which may generate perturbations and also significant non-Gaussianities. Among such possibilities, for example, the curvaton mechanism [54, 55, 56, 57, 58, 59, 72, 61], has been proposed and some of their observational consequences including issues of the non-Gaussianities have also been investigated.

As addressed in the introduction, in the multiple-field scenarios, curvature perturbation can generally evolve after the horizon-exiting. Detectable non-Gaussianities can be produced when the curvature perturbation is generated from the entropy perturbations at the end of inflation [67, 32], or during reheating process [68, 69, 70]. Moreover, in string inspired inflationary models, other string effects, for example, cosmic string effect should also be considered [74, 75].

In this work, we studied the non-linearity of primordial perturbations measured by three-point functions. Indeed, the deviation from a Gaussian distribution may also be induced by higher-order correlation functions. It is interesting to go forward to find the exact fourth-order action of curvature perturbation and investigate the non-Gaussianities from the four-point functions and trispectra [62, 63, 64, 65, 66].

Finally, in this work we only considered the case of $c_s \approx 1$, it would be interesting to extend the current result to the case of an arbitrary, especially a small c_s , which would be another source of large non-Gaussianities. On the other hand, the studies of multiple DBI-inflation [45, 46] show that in the limit of $c_s \ll 1$, the multi-field effects are suppressed by c_s , and thus the multi-field models reduce to a effective single-field model. We would like to leave this for a future investigation.

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A. General Structure of the Expansion of the Action

In order to expand the action (3.2) to the third-order of Q^I , we need the expansions for $P(X, \phi^I)$ and $E_{ij}E^{ij} - E^2$. Firstly, we expand P and X as

$$\begin{aligned} P &= P^{(0)} + P^{(1)} + P^{(2)} + P^{(3)} + \dots, \\ X &= X^{(0)} + X^{(1)} + X^{(2)} + X^{(3)} + \dots, \end{aligned} \quad (\text{A.1})$$

where $P^{(n)}$ and $X^{(n)}$ are $\mathcal{O}(Q^n)$ pieces of P and X respectively. From (3.4)-(3.5), we have

$$\begin{aligned} X^{(1)} &= \dot{\phi}_I \dot{Q}^I - \frac{P_{,X} X}{H} \dot{\phi}_I \dot{Q}^I, \\ X^{(2)} &= \frac{1}{2} \dot{Q}_I \dot{Q}^I - \dot{\phi}_I \partial^i \beta_1 \partial_i Q^I - \frac{P_{,X}}{H} \dot{\phi}_I \dot{\phi}_J \dot{Q}^I Q^J + \frac{3P_{,X}^2 X}{4H^2} \dot{\phi}_I \dot{\phi}_J \dot{Q}^I Q^J - \frac{1}{2a^2} G_{IJ} \partial_i Q^I \partial_i Q^J, \\ X^{(3)} &= -\frac{P_{,X}}{2H} \dot{\phi}_I \dot{Q}^I \left(\dot{Q}_I \dot{Q}^I - 2\dot{\phi}_I \partial^i \beta_1 \partial_i Q^I \right) + \frac{3P_{,X}^2}{4H^2} \dot{\phi}_I \dot{\phi}_J \dot{\phi}_K \dot{Q}^I Q^J Q^K - \frac{P_{,X}^3 X}{2H^3} \dot{\phi}_I \dot{\phi}_J \dot{\phi}_K \dot{Q}^I Q^J Q^K - \dot{Q}_I \partial^i \beta_1 \partial_i Q^I. \end{aligned} \quad (\text{A.2})$$

Expanding P to the third-order, we have

$$\begin{aligned} P^{(1)} &= P_{,X} X^{(1)} + P_{,I} Q^I, \\ P^{(2)} &= P_{,X} X^{(2)} + \frac{1}{2} P_{,XX} (X^{(1)})^2 + \frac{1}{2} P_{,IJ} Q^I Q^J + P_{,XI} X^{(1)} Q^I, \\ P^{(3)} &= P_{,X} X^{(3)} + P_{,XX} X^{(1)} X^{(2)} + P_{,XI} X^{(2)} Q^I \\ &\quad + \frac{1}{6} P_{,XXX} (X^{(1)})^3 + \frac{1}{2} P_{,XXI} (X^{(1)})^2 Q^I + \frac{1}{2} P_{,XIJ} X^{(1)} Q^I Q^J + \frac{1}{6} P_{,IJK} Q^I Q^J Q^K. \end{aligned} \quad (\text{A.3})$$

The first integral in the action (3.2) can be expanded as

$$\begin{aligned} S_A &\equiv \int dt d^3x \sqrt{h} N P \\ &= \int dt d^3x a^3 (1 + \alpha_1 + \dots) \left(P^{(0)} + P^{(1)} + P^{(2)} + P^{(3)} + \dots \right) \\ &= \int dt d^3x a^3 \left(P^{(0)} + P^{(1)} + \alpha_1 P^{(0)} + P^{(2)} + \alpha_1 P^{(1)} + P^{(3)} + \alpha_1 P^{(2)} + \dots \right). \end{aligned} \quad (\text{A.4})$$

Therefore the second and the third-order pieces of S_A are

$$S_A^{(2)} = \int dt d^3x a^3 \left(P^{(2)} + \alpha_1 P^{(1)} \right), \quad (\text{A.5})$$

$$S_A^{(3)} = \int dt d^3x a^3 \left(P^{(3)} + \alpha_1 P^{(2)} \right), \quad (\text{A.6})$$

respectively, where $P^{(n)}$'s are given by (A.3).

Now we consider the second integral in (3.2), which we denote as S_B . First we may expand $E_{ij}E^{ij} - E^2$ as

$$\begin{aligned}\Pi &\equiv \frac{1}{2}(E_{ij}E^{ij} - E^2) = \Pi^{(0)} + \Pi^{(1)} + \Pi^{(2)} + \dots \\ &= -3H^2 + \frac{2H}{a^2}\partial^2\beta_1 + \left(\frac{1}{2a^4}\partial_i\partial_j\beta_1\partial_i\partial_j\beta_1 - \frac{1}{2a^4}(\partial^2\beta_1)^2\right) + \dots,\end{aligned}\tag{A.7}$$

then

$$\begin{aligned}S_B &= \int dt d^3x \frac{a^3}{N} \Pi \\ &= \int dt d^3x a^3 \left[\Pi^{(0)} + (\Pi^{(1)} - \alpha_1 \Pi^{(0)}) + (\Pi^{(2)} - \alpha_1 \Pi^{(1)} + \alpha_1^2 \Pi^{(0)}) + (-\alpha_1 \Pi^{(2)} + \alpha_1^2 \Pi^{(1)} - \alpha_1^3 \Pi^{(0)}) + \dots \right],\end{aligned}\tag{A.8}$$

The second and third-order pieces of the second integral in the coupled action (3.2) now read

$$S_B^{(2)} = \int dt d^3x a^3 (\Pi^{(2)} - \alpha_1 \Pi^{(1)} + \alpha_1^2 \Pi^{(0)}),\tag{A.9}$$

$$S_B^{(3)} = \int dt d^3x a^3 (-\alpha_1 \Pi^{(2)} + \alpha_1^2 \Pi^{(1)} - \alpha_1^3 \Pi^{(0)}),\tag{A.10}$$

where $\Pi^{(n)}$'s are given by (A.7).

Finally, the second and third-order pieces of the effective action are

$$S^{(2)} = S_A^{(2)} + S_B^{(2)} = \int dt d^3x a^3 \left(P^{(2)} + \alpha_1 P^{(1)} + \Pi^{(2)} - \alpha_1 \Pi^{(1)} + \alpha_1^2 \Pi^{(0)} \right),\tag{A.11}$$

$$S^{(3)} = S_A^{(3)} + S_B^{(3)} = \int dt d^3x a^3 \left(P^{(3)} + \alpha_1 P^{(2)} - \alpha_1 \Pi^{(2)} + \alpha_1^2 \Pi^{(1)} - \alpha_1^3 \Pi^{(0)} \right).\tag{A.12}$$

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